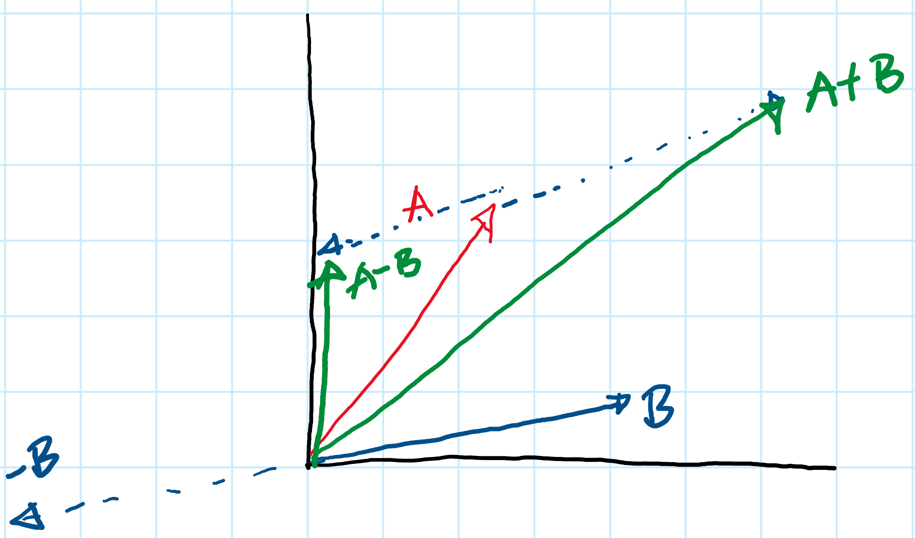
Phasor Operations



Tail to Tip addition

$$\bar{A} - \bar{B} = A + (-\bar{B})$$

$$\bar{A} = A\angle\alpha^\circ$$

$$\bar{B} = B\angle\beta^\circ$$

$$\bar{A}\bar{B} = AB\angle(\alpha+\beta)^\circ$$

$$\bar{A}^* = A\angle-\alpha^\circ \qquad \bar{B}^* = B\angle-\beta$$

$$\bar{A}\bar{A}^* = AA\angle(\alpha-\alpha)^\circ = A^2$$

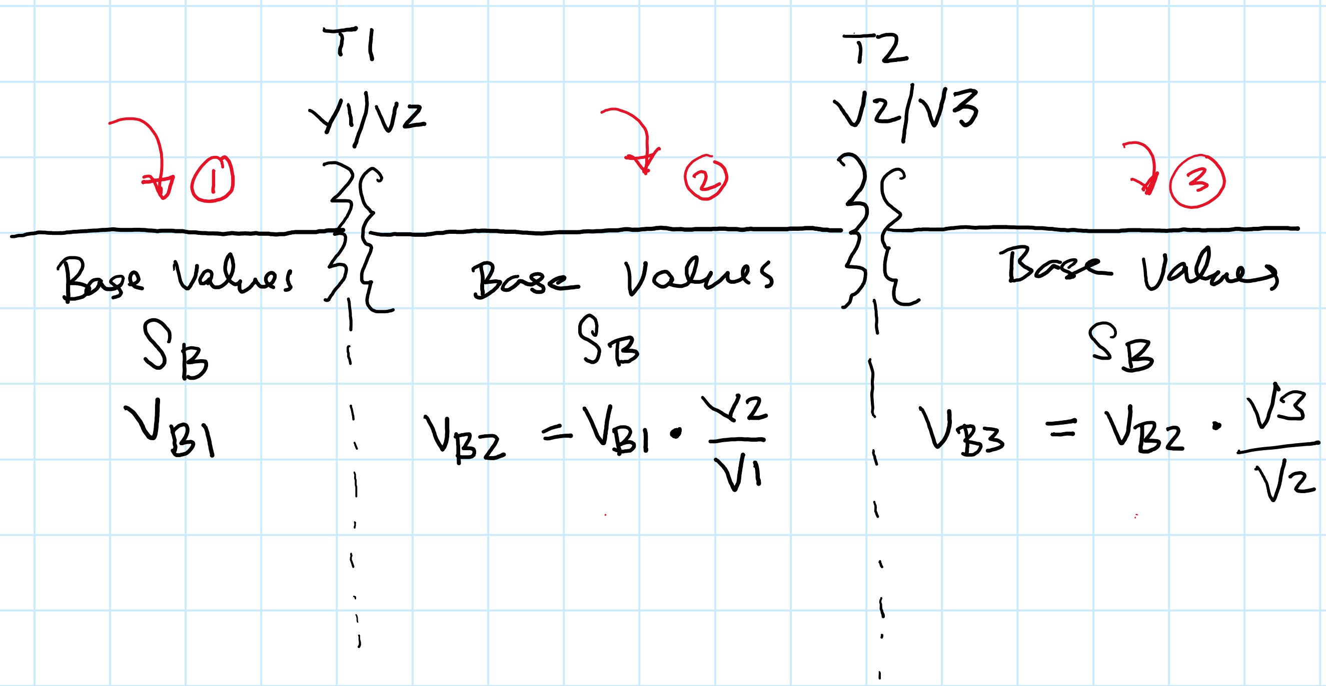
$$\frac{\bar{A}}{\bar{B}} = \frac{A}{B}\angle(\alpha-\beta)^\circ$$

Per Unit Method

Per Unit Quantity = actual quantity / base quantity
Actual quantities can be scaler or complex
Percent Quantity = per unit × 100%

STEP ① Select the three-phase base power (S_b) and the phase-to-phase base voltage (V_b) values. Base power is the same for the whole system. Base voltage needs to be adjusted when crossing transformers according to transformer ratio

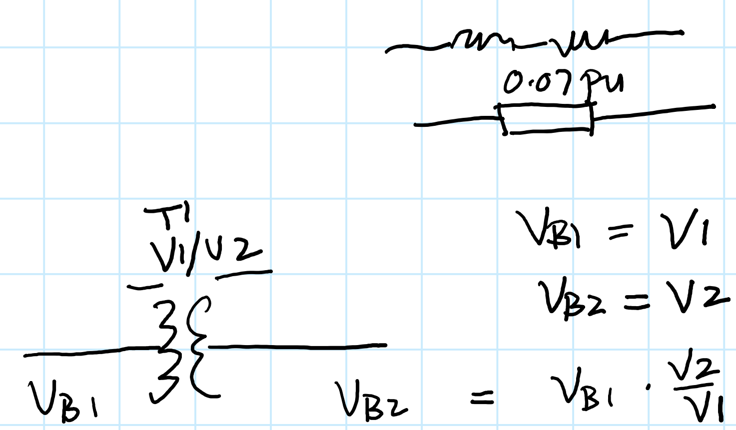
STEP ② Calculate base current (I_b) and base impedance (Z_b) for each voltage level.



$n = 1, 2, 3, \text{etc.}$

$$I_{Bn} = \frac{S_B}{\sqrt{3}\, V_{Bn}}$$

$$Z_{Bn} = \frac{V_{Bn}/\sqrt{3}}{I_{Bn}} = \frac{V_{Bn}^2}{S_B}$$



$V_{B1} = V1$

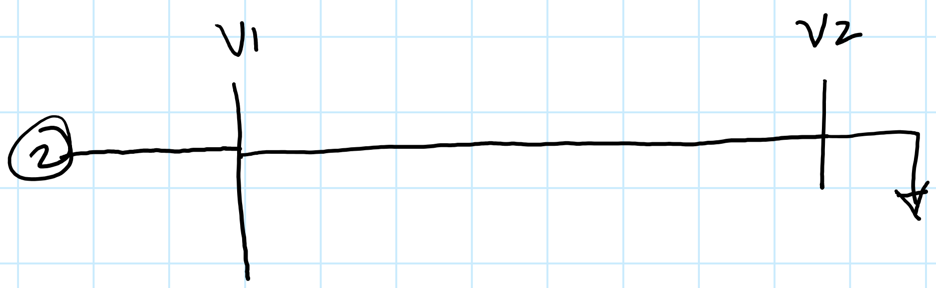
$V_{B2} = V2$

$V_{B2} = V_{B1} \cdot \frac{V2}{V1}$

$$V_{B1}(pu) = \frac{V_{B1}}{V_{B1}} = 1\ pu$$

$$V_{B2}(pu) = \frac{V_{B2}}{V_{B2}} = 1\ pu$$

Example power flow on 69 kV Transmission Line



$S = P + jQ$

Sending End

$V_1 = 74.29 \angle 6.05^\circ$ kV $\qquad \bar{S} = 62.65 + j58.26$ MVA

Base Voltage = 69 kV

Base Power = 100 MVA

$$V_1 = \frac{74.29 \angle 6.05^\circ \text{ kV}}{69 \text{ kV}} = 1.077 \text{ pu}$$

$$S = \frac{62.65 + j58.26 \text{ MVA}}{100 \text{ MVA}} = 0.6265 + j0.5826 \text{ pu}$$

Changing Base

$$Z_{pu}^{OLD} = \frac{Z}{Z_{BASE}^{OLD}} = Z \cdot \frac{S_{BASE}^{OLD}}{(V_{BASE}^{OLD})^2}$$

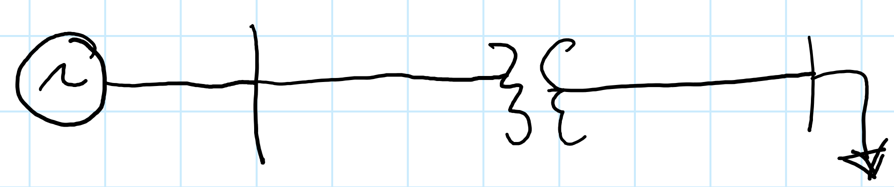
$$Z_{pu}^{NEW} = \frac{Z}{Z_{BASE}^{NEW}} = Z \cdot \frac{S_{BASE}^{NEW}}{(V_{BASE}^{NEW})^2}$$

$$Z_{pu}^{NEW} = Z \cdot \frac{S_{BASE}^{NEW}}{(V_{BASE}^{NEW})^2}$$

$$Z = Z_{pu}^{OLD} \cdot \frac{(V_{BASE}^{OLD})^2}{S_{BASE}^{OLD}}$$

$$Z_{pu}^{NEW} = Z_{pu}^{OLD} \cdot \frac{S_{BASE}^{NEW}}{S_{BASE}^{OLD}} \cdot \left(\frac{V_{BASE}^{OLD}}{V_{BASE}^{NEW}}\right)^2$$ ✓✓

EXAMPLE: Using 100MVA base, 110kV base (high side), find the following in per unit



25MVA
4kV
$X_s = 25\%$

30MVA
4.2/115kV
operating at 3.9kV Tap
$X_T = 10\%$

$S_B = 100$ MVA
$V_{BASE}\ H = 110$kV

$$\frac{V_{BASE}\ L}{V_{BASE}\ H} = \frac{V_L}{V_H}$$

$$\frac{V_{BASE}\ L}{110\text{kV}} = \frac{3.9\text{kV}}{115\text{kV}}$$

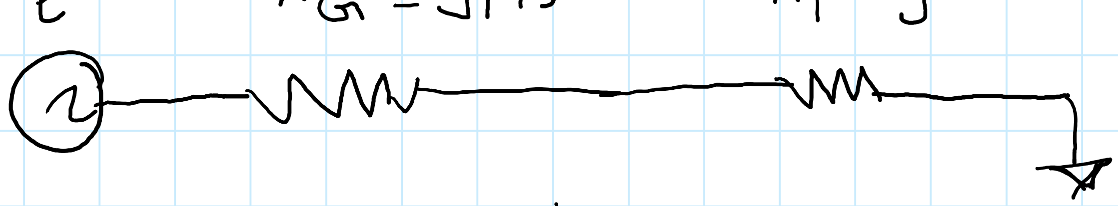
$V_{BASE}\ L = 3.9 \times 110/115$
$= 3.73$kV

$$X_G = \frac{25}{100}\left(\frac{100}{25}\right)\left(\frac{4\text{kV}}{3.73\text{kV}}\right)^2 = 1.15 \text{ pu}$$

$$X_{TH} = \frac{10}{100}\left(\frac{100}{30}\right)\left(\frac{115\text{kV}}{110\text{kV}}\right)^2 = 0.3643 \text{ pu}$$

$$X_{TL} = \frac{10}{100}\left(\frac{100}{30}\right)\left(\frac{3.9}{3.73}\right)^2 = 0.3643 \text{ pu}$$

E $\quad X_G = j1.15 \quad X_T = j0.364$



E $\quad X_{TOTAL} = j1.514$