N - number of phasors

α - angle shift between phasors

"a" operator $= 1\angle\alpha$

$\alpha = \frac{360^\circ}{N}$ $\quad a = 1\angle\alpha$

$-\alpha \cdot n$; n = phase sequence network number

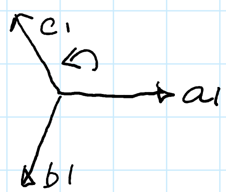
Sequence Domain

Sequence #1

$a1$ @ zero degree ref

$b1$ angle $= -\alpha \cdot n = (-1\angle 120) \cdot 1 = -1\angle 120$

$C1$ angle $= -\alpha \cdot n = (-1\angle 120) \cdot 1 = -1\angle 120$

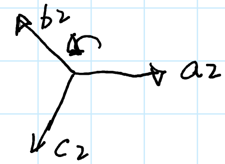


Sequence #2

$a2$ @ zerodegree

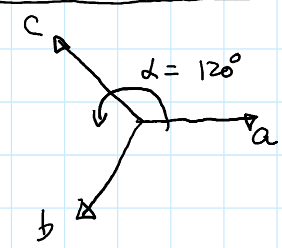
$b2$ angle $= -\alpha \cdot n = -(1\angle 120) \times 2 = -1\angle 240$

$C2$ angle $= -\alpha \cdot n = (-1\angle 120)\, 2 = -1\angle 240$



3-Phase System Example

Physical Domain



$N = 3$

$n = 0, 1, 2 \ldots N-1$

$\alpha = 360/3 = 120^\circ$

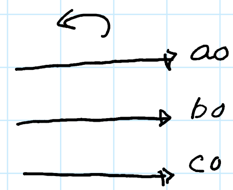
Sequence Domain

Sequence # 0

Say $a0$ is at 0° reference

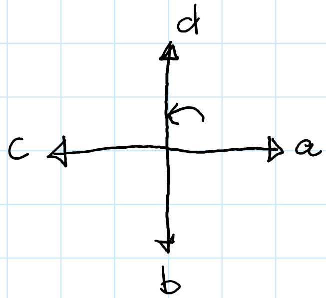
$b0$ angle $= -\alpha \cdot n = (-1\angle 120) \times 0$
$= 0^\circ$

$c0$ angle $= -\alpha \cdot n = (-1\angle 120) \times 0$
$= 0^\circ$



4-Phase System Example

Physical Domain



$N = 4$

$n = 0, 1, 2, 3$

$\alpha = 360/4 = 90^\circ$

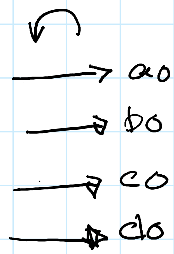
Sequence Domain

Sequence #0

a_0 angle $= 0^\circ$

$b_0, c_0, d_0 = \alpha \cdot n$

$= -1\angle 90^\circ \times 0 = 0^\circ$



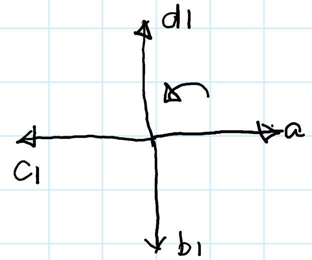
Sequence #1

a_1 @ 0°

$b_1 = 0 + (-90 \times 1) = -90^\circ$

$c_1 = -90^\circ + (-90 \times 1) = -180$

$d_1 = -180 + (-90 \times 1) = -270^\circ$



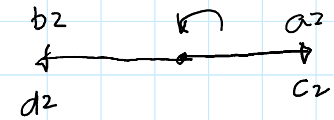
Sequence #2

a_2 @ 0°

$b_2 = 0^\circ + (-90 \times 2) = -180^\circ$

$c_2 = -180^\circ + (-90 \times 2) = -360^\circ$

$d_2 = -360 + (-90 \times 2) = -180^\circ$



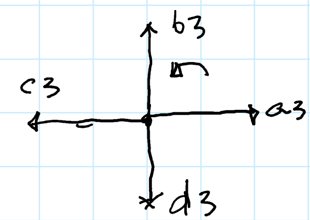
Sequence #3

a_3 @ 0°

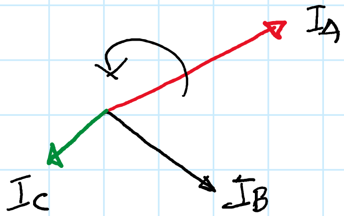
$b_3 = 0 + (-90 \times 3) = -270^\circ$

$c_3 = -270^\circ + (-90 \times 3) = -540^\circ = -180$

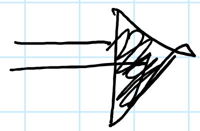
$d_3 = -540 + (-90 \times 3) = -810 = -90$



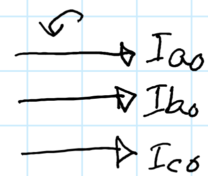
Decomposition of an Unbalanced Set of Phasors



Physical System



$I_{a0} = I_{b0} = I_{c0}$

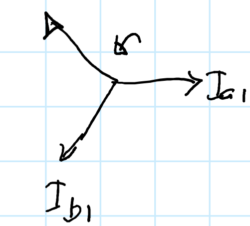


Zero Sequence

+

$I_{b1} = a^2 I_{a1}$

$I_{c1} = a I_{a1}$

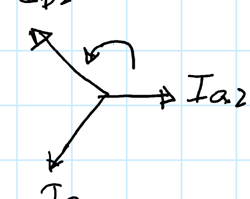


Positive Sequence

+

$I_{b2} = a I_{a2}$

$I_{c2} = a^2 I_{a2}$



Negative Sequence

Sequence Domain

$a = 1\angle\alpha = 1\angle 120^\circ$

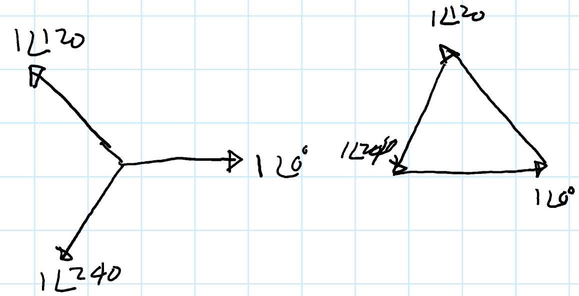
$\alpha = 120^\circ$

Phase Quantities as a Function of Symmetrical Components

$$
\begin{aligned}
I_A &= I_{A0} + I_{A1} + I_{A2} &&= I_{A0} + I_{A1} + I_{C1} &&= I_0 + I_1 + I_2 \\
I_B &= I_{B0} + I_{B1} + I_{B2} &&= I_{A0} + a^2 I_{A1} + a\, I_{A2} &&= I_0 + a^2 I_1 + a I_2 \\
I_C &= I_{C0} + I_{C1} + I_{C2} &&= I_{A0} + a I_{A1} + a^2 I_{A2} &&= I_0 + a I_1 + a^2 I_2
\end{aligned}
$$

We could drop/eliminate subscript "A" noting that we are using phase A as reference

Sequence Quantities as a function of Zero Sequence



$$I_A = I_0 + I_1 + I_2$$
$$I_B = I_0 + a^2 I_1 + a I_2$$
$$I_C = I_0 + a I_1 + a^2 I_2$$

$$I_A + I_B + I_C = (I_0 + I_0 + I_0) + I_1(\cancelto{0}{1 + a^2 + a}) + I_2(\cancelto{0}{1 + a + a^2})$$
$$3I_0 = I_A + I_B + I_C$$

$$I_0 = 1/3(I_A + I_B + I_C)$$

Sequence Quantities as a function of Positive Sequence

$$I_A = I_0 + I_1 + I_2$$
$$(I_B = I_0 + a^2 I_1 + a I_2) \times a$$
$$(I_C = I_0 + a I_1 + a^2 I_2) \times a^2$$

$$I_A + aI_B + a^2 I_C = I_0(\cancel{1 + a + a^2}) + 3I_1 + I_2(\cancel{1 + a^2 + a})$$
$$3I_1 = I_A + aI_B + a^2 I_C$$
$$I_1 = 1/3(I_A + aI_B + a^2 I_C)$$

Sequence Quantities as a Function of Negative Sequence

$$I_A = I_0 + I_1 + I_2$$

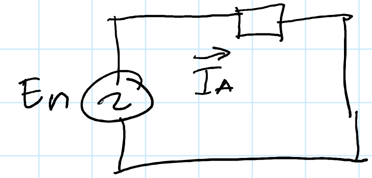
$$(I_B = I_0 + a^2 I_1 + a I_2) \times a^2$$

$$(I_C = I_0 + a I_1 + a^2 I_2) \times a$$

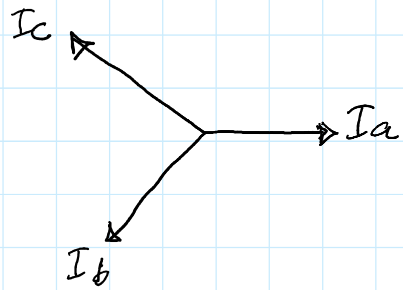
$$I_A + a^2 I_B + a I_C = I_0 \cancel{(1 + a^2 + a)} + I_1 \cancel{(1 + a^4 + a^2)} + I_2 (1 + a^3 + a^3)$$

$$3 I_2 = I_A + a^2 I_B + a I_C$$

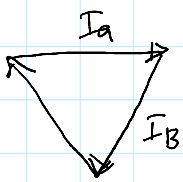
$$I_2 = 1/3 (I_A + a^2 + a I_C)$$



Balanced System : Sequence Behaviour



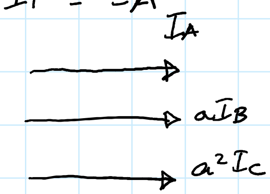
$$3I_0 = I_A + I_B + I_C = 0$$



$$3I_1 = I_A + aI_B + a^2 I_C \qquad I_A = I_1 = \frac{E_n}{Z_1}$$

$$3I_1 = 3I_A$$

$$I_1 = I_A$$



$I_B = 1\angle -120$

$I_C = 1\angle 120$

$aI_B = 1\angle -120 \times 1\angle 120 = 1\angle(-120+120)$

$a^2 I_2 = 1\angle 120 \times 1\angle 240 = 1\angle 360^\circ$

$$3I_2 = I_A + a^2 I_B + aI_C = 0$$

$a^2 I_B = 1\angle 240 \times 1\angle -120$

$= 1\angle 120$

$aI_c = 1\angle 120 \times \angle 120$

$= 1\angle 240$