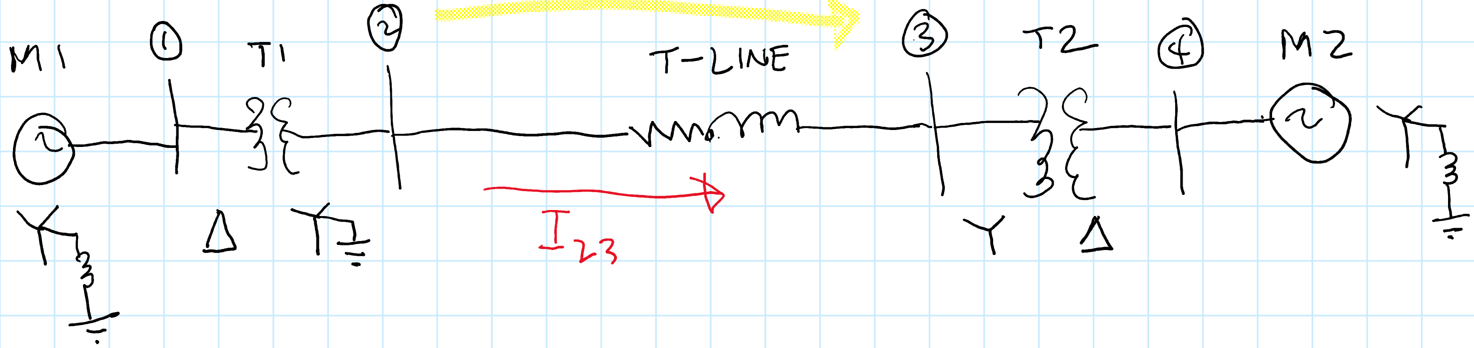
SERIES OR OPEN FAULTS SAMPLE CALCULATIONS



Machines M_1 and M_2 : 100 MVA, 20kV $X_d'' = X_1 = X_2 = 20\%$; $X_0 = 4\%$; $X_n = 5\%$

Transformers T_1 and $T2$: 100 MVA, 20Δ/345 Y kV ; $X = 8\%$

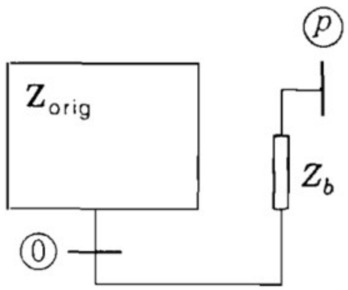
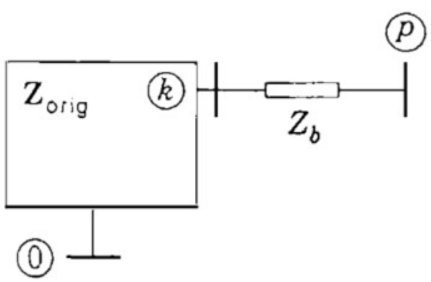
T-LINE : $X_1 = X_2 = 15\%$; $X_0 = 50\%$

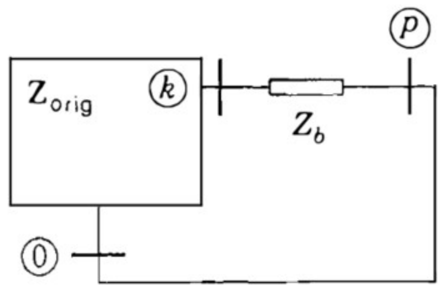
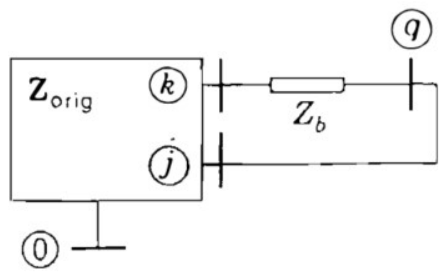
REQUIRED: Determine the change in voltage at bus ③ when the transmission line undergoes (a) one-open conductor fault and (b) a two-conductor open fault along its span between buses ② and ③. Consider that Machine # 2 is a motor drawing a load equivalent to 60 MVA at 80% power factor lagging and nominal 345kV voltage at Bus ③.

Steps to calculate for the change in bus voltage due to Open Faults

1. Obtain pre-fault information (load current being delivered)
2. Draw the sequence diagrams (positive, negative, zero-sequence impedance diagrams)
3. Obtain the Bus Impedance Matrix (Zbus matrix) for all three networks
4. Obtain equations for the (+/-/0) sequence impedances as seen from the point of open phase/line conductor
5. Calculate the (+/-/0) sequence voltage drops (equation depends on the type of fault whether one-phase or two-phase open).
6. Calculate the change in the voltage (delta-V) for all three sequences
7. Calculate the final voltage by adding the delta-V and the pre-fault voltage

Modification of existing $\mathbf{Z}_{bus}$

Case	Add branch $\mathbf{Z}_b$ from	$\mathbf{Z}_{bus\,(new)}$
1	Reference node to new bus (p) 	$\begin{array}{cc} & \begin{array}{cc}\hspace{6em} & p\end{array} \\ \begin{array}{c} \\ \\ p\end{array} & \left[\begin{array}{c\|c} \mathbf{Z}_{orig} & \begin{matrix}0\\ \vdots\\ 0\end{matrix} \\ \hline 0 \ \cdots \ 0 & Z_b \end{array}\right]\end{array}$
2	Existing bus (k) to new bus (p) 	$\begin{array}{cc} & \begin{array}{cc}\hspace{3em} k & \hspace{3em} p\end{array} \\ \begin{array}{c} k \\ \\ p\end{array} & \left[\begin{array}{c\|c} \mathbf{Z}_{orig} & \text{col. } k \\ \hline \text{row } k & Z_{kk} + Z_b \end{array}\right]\end{array}$

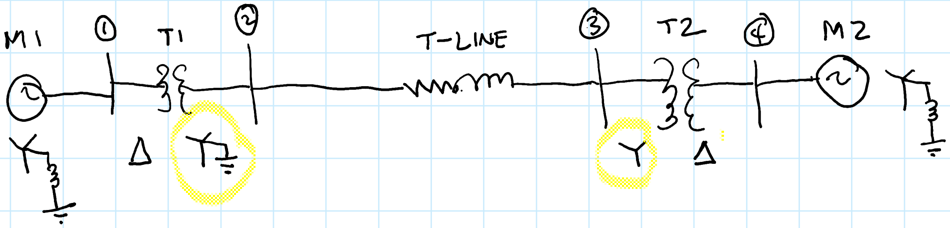
3	Existing bus k to reference node  (Node p is temporary.)	• Repeat Case 2 and • Remove row p and column p by Kron reduction
4	Existing bus j to existing bus k  (Node q is temporary.)	• Form the matrix $\begin{array}{cc} & \begin{array}{cc} \hphantom{\text{row } j - \text{row } k} & q \end{array} \\ \begin{array}{c} \\ q \end{array} & \left[\begin{array}{c\|c} \mathbf{Z}_{\text{orig}} & \text{col. } j - \text{col. } k \\ \hline \text{row } j - \text{row } k & Z_{\text{th},jk} + Z_b \end{array}\right] \end{array}$ where $Z_{\text{th},jk} = Z_{jj} + Z_{kk} - 2Z_{jk}$ and • Remove row q and column q by Kron reduction

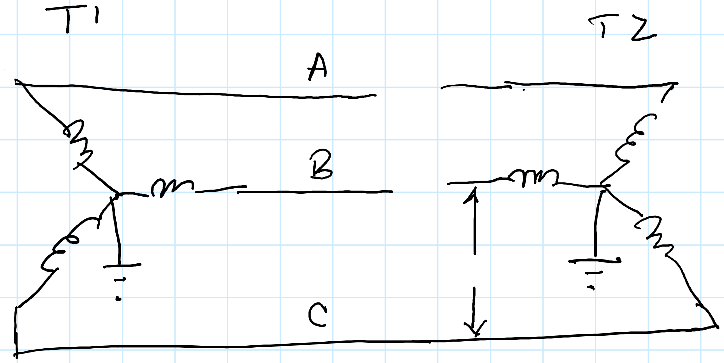
STEP # 1

Calculate for I_{23} (the current flowing from Bus ② to Bus ③.

Choose 100MVA as Power Base :

$$I_{23} = \frac{60/100 \angle -\cos^{-1} 0.8}{1.0 \angle 0^\circ} = \boxed{0.6 \angle -36.87^\circ} \text{ pu}$$





Machines M_1 and M_2 : 100 MVA, 20 kV $X_d'' = X_1 = X_2 = 20\%$; $X_0 = 4\%$; $X_n = 5\%$

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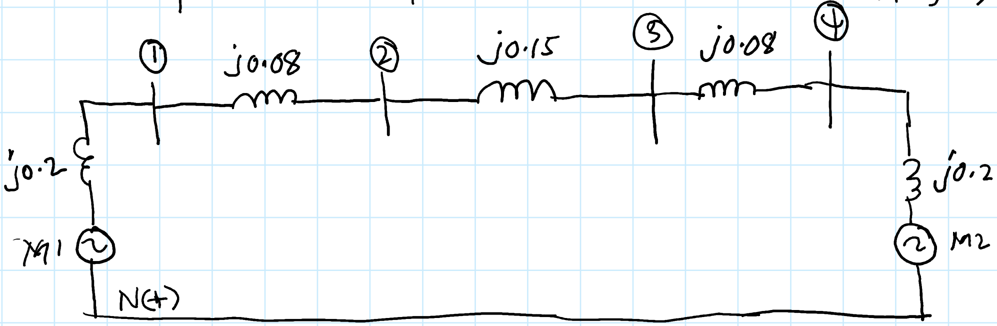
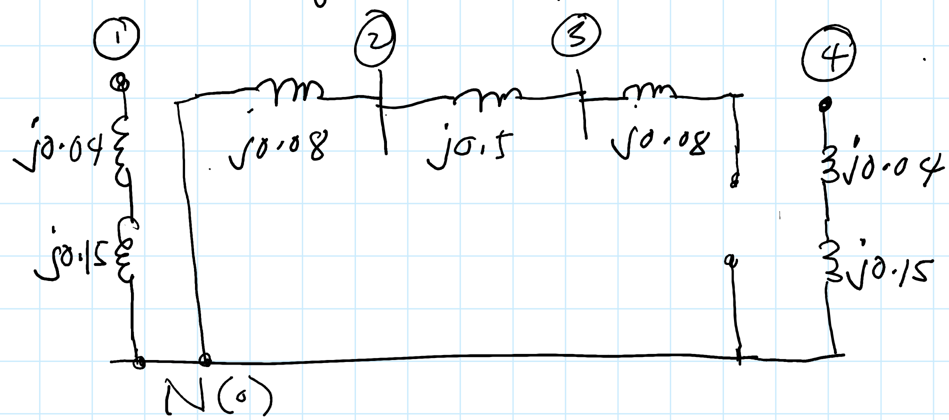
STEP #2 : Draw the Sequence Impedance Networks

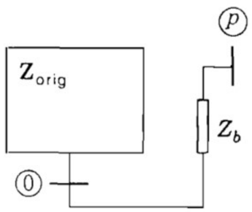
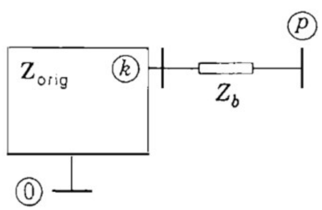
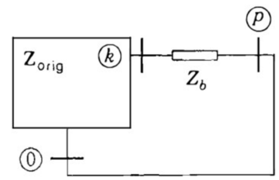
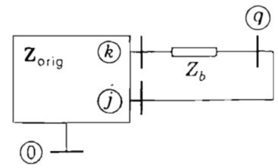
Positive Sequence Impedance (Negative Sequence Impedance is the same but no source voltage)



Zero Sequence Impedance



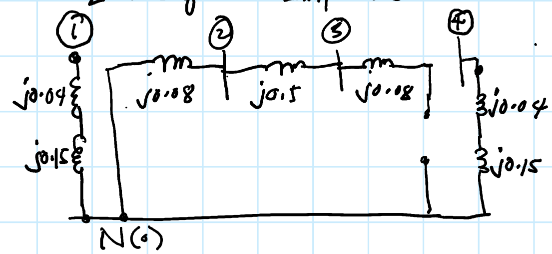
Modification of existing $\mathbf{Z}_{bus}$

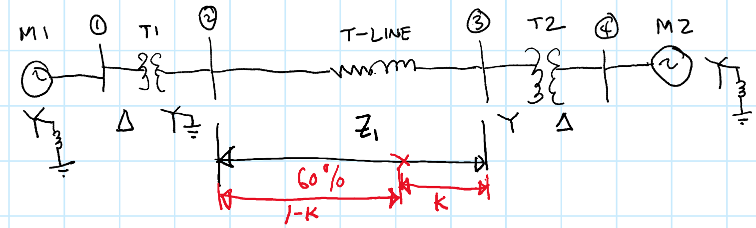
Case	Add branch $\mathbf{Z}_b$ from	$\mathbf{Z}_{bus\,(new)}$
1	Reference node to new bus (p) 	$\begin{array}{cc} & \begin{matrix} \qquad\qquad & (p) \end{matrix} \\ (p) & \left[\begin{array}{c\|c} \mathbf{Z}_{orig} & \begin{matrix}0\\ \vdots \\ 0\end{matrix} \\ \hline 0 \cdots 0 & Z_b \end{array}\right] \end{array}$
2	Existing bus (k) to new bus (p) 	$\begin{array}{cc} & \begin{matrix} (k) & \qquad & (p) \end{matrix} \\ \begin{matrix} (k) \\ (p) \end{matrix} & \left[\begin{array}{c\|c} \mathbf{Z}_{orig} & \text{col. } k \\ \hline \text{row } k & Z_{kk} + Z_b \end{array}\right] \end{array}$
3	Existing bus (k) to reference node  (Node (p) is temporary.)	• Repeat Case 2 and • Remove row p and column p by Kron reduction
4	Existing bus (j) to existing bus (k)  (Node (q) is temporary.)	• Form the matrix $\begin{array}{cc} & \begin{matrix} \qquad\qquad & (q) \end{matrix} \\ (q) & \left[\begin{array}{c\|c} \mathbf{Z}_{orig} & \text{col. } j - \text{col. } k \\ \hline \text{row } j - \text{row } k & Z_{th,\,jk} + Z_b \end{array}\right] \end{array}$ where $Z_{th,\,jk} = Z_{jj} + Z_{kk} - 2Z_{jk}$ and • Remove row q and column q by Kron reduction

ZBUS MATRIX

Positive Sequence Impedance (Negative Sequence Impedance is the Same but no Source Voltage)

Zero Sequence Impedance





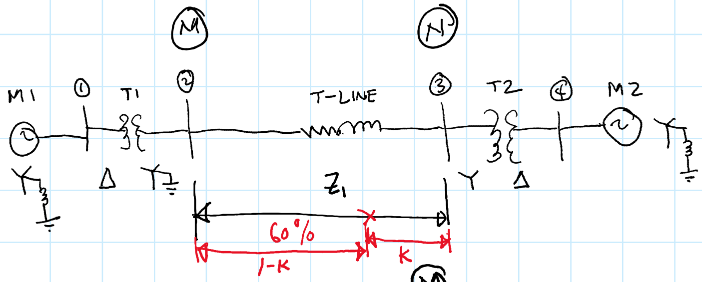
$$\sum_{i=1}^{N} \sum_{j=1}^{N} Y'_{ij} = \frac{Y_{ij} - Y_{ik} Y_{jk}}{Y_{kk}}$$

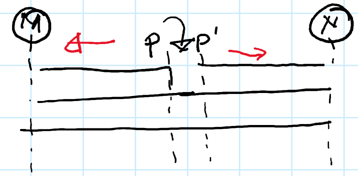
Positive & Negative Seq.

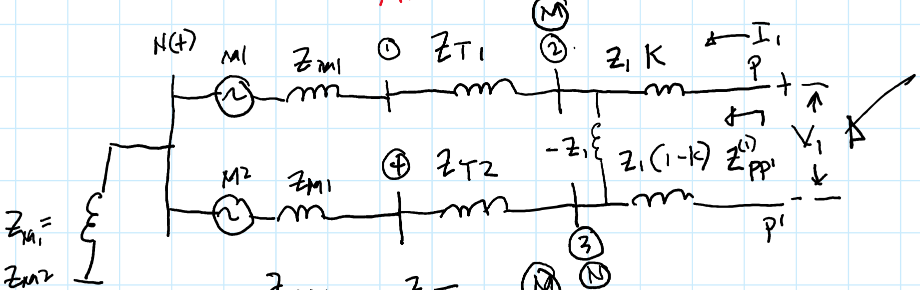
R\C		1	2	3	4
	Bus	1	2	3	4
1	1	0.1437	0.1211	0.0789	0.0563
2	2	0.1211	0.1696	0.1104	0.0789
3	3	0.0789	0.1104	0.1696	0.1211
4	4	0.0563	0.0789	0.1211	0.1437

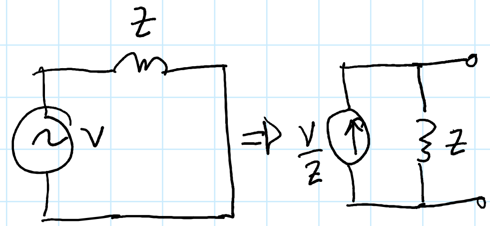
Zero Sequence

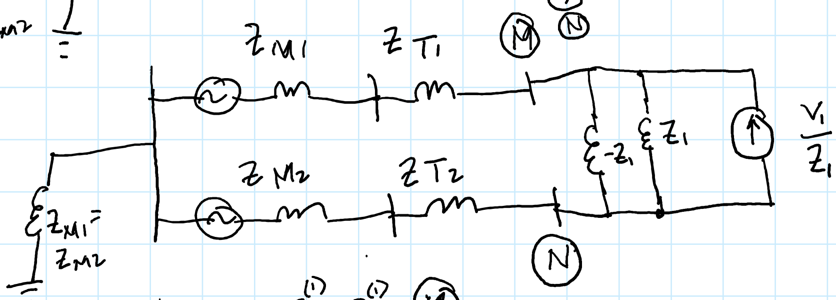
R\C		1	2	3	4
	Bus	1	2	3	4
1	1	0.19	0	0	0
2	2	0	0.08	0.08	0
3	3	0	0.08	0.58	0
4	4	0	0	0	0.19

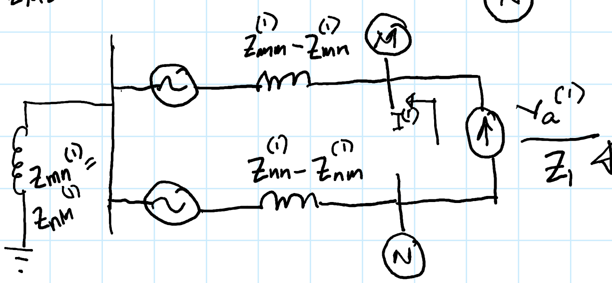












line impedance

$$Z_{pp'} = KZ_1 + \frac{(-Z_1)(Z_{TH,MN})}{(-Z_1)+(Z_{TH,MN})} + \overset{-KZ_1}{(1-K)Z_1} =$$

$$= \frac{\cancel{KZ_1(Z_{TH,MN}) - KZ_1^2} + \cancel{(-Z_1)(Z_{TH,MN}) + Z_1(Z_{TH,MN})} - Z_1^2 \cancel{- KZ_1(Z_{TH,MN}) + KZ_1^2}}{(Z_{TH,MN}) - (Z_1)}$$

$$Z_{pp'} = \frac{-(Z_1)^2}{(Z_{TH,MN}) - (Z_1)}$$

Impedance as seen from the faulted point or breaking point of line conductor (equation applies to positive, negative, zero seq.)

$$Z_{pp'}^{(1)} = \frac{-(Z_1)^2}{(Z_{TH,MN}) - (Z_1)} = \frac{-(j0.15)^2}{(j0.1696) + (j0.1696) - 2(j0.1104) - (j0.15)} = \boxed{j0.7120 \text{ pu}}$$ (positive & negative seq. impedance)

Z_1 = conductor or transmission line Impedance

$Z_{TH,MN}$ = equivalent Impedance behind bus M and bus N

Z_{MM} = self impedance of bus M

Z_{MN} = Transfer Impedance between bus M and bus N

Z_{NN} = Self Impedance of bus N

$Z_{NM} = Z_{MN}$ = Transfer Impedance between bus N and bus M

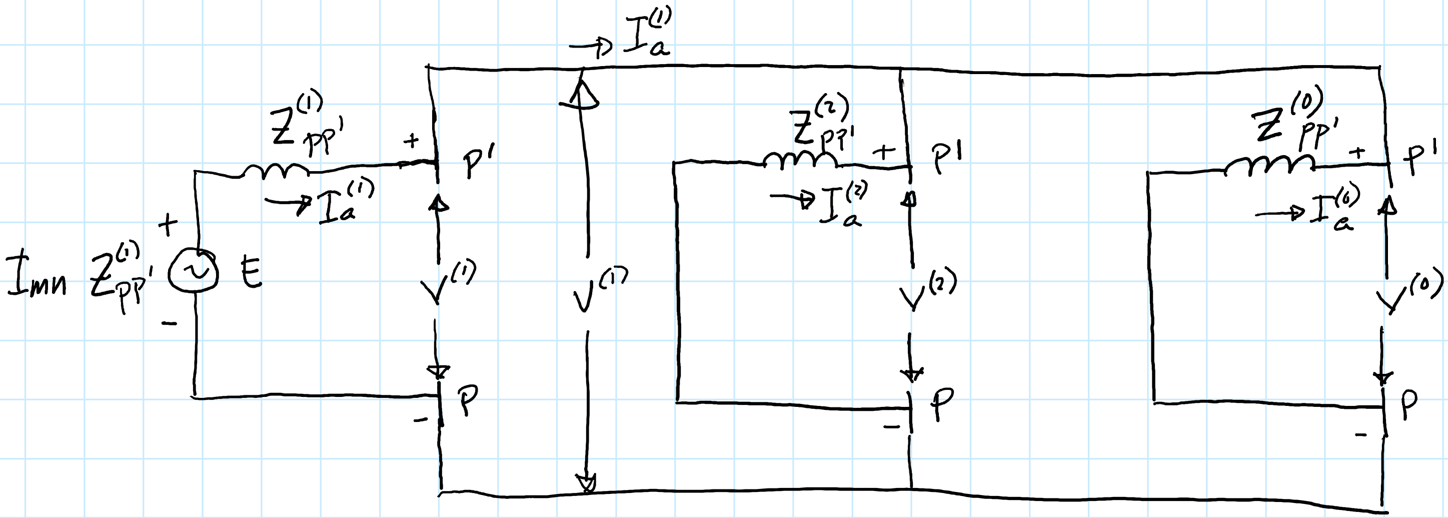
$$Z_{TH,MN} = (Z_{MM} - Z_{MN}) + (Z_{NN} - Z_{NM})$$

$$Z_{TH,MN} = Z_{MM} + Z_{NN} - 2(Z_{MN})$$

$Z_{MM} = Z_{22}$; $Z_{NN} = Z_{33}$; $Z_{MN} = Z_{23}$; $Z_{NM} = Z_{32}$

$$Z_{pp'}^{(0)} = \frac{-(Z_1)^2}{(Z_{TH,MN}) - (Z_1)} = \frac{-(j0.5)^2}{(j0.08) + (j0.58) - 2(j0.08) - (j0.5)}$$

$$Z_{pp'}^{(0)} = \frac{-(j0.5)^2}{0} = \infty$$ (infinite Impedance for zero sequence)



$$I_a^{(1)} = \frac{E}{Z_{TH}} = \frac{I_{mn} Z^{(1)}_{pp'}}{Z^{(1)}_{pp'} + \dfrac{Z^{(2)}_{pp'} Z^{(0)}_{pp'}}{Z^{(2)}_{pp'} + Z^{(0)}_{pp'}}} = \frac{I_{mn} Z^{(1)}_{pp'} (Z^{(2)}_{pp'} + Z^{(0)}_{pp'})}{Z^{(0)}_{pp'} Z^{(1)}_{pp'} + Z^{(1)}_{pp'} Z^{(2)}_{pp'} + Z^{(2)}_{pp'} Z^{(0)}_{pp'}}$$

$$V_a^{(1)} = V_a^{(2)} = V_a^{(0)} = \frac{I_a^{(1)} Z^{(2)}_{pp'} \cdot Z^{(0)}}{Z^{(2)}_{pp'} + Z^{(0)}_{pp'}} = \frac{I_{mn} Z^{(1)}_{pp'} \left[Z^{(2)}_{pp'} + Z^{(0)}_{pp'} \right] \left[Z^{(2)}_{pp'} Z^{(0)}_{pp'} \right]}{\left[Z^{(2)}_{pp'} + Z^{(0)}_{pp'} \right] \left[Z^{(0)}_{pp'} Z^{(1)}_{pp'} + Z^{(1)}_{pp'} Z^{(2)}_{pp'} + Z^{(2)}_{pp'} Z^{(0)}_{pp'} \right]}$$

$$V_a^{(1)} = V_a^{(2)} = V_a^{(0)} = \frac{I_{mn} \left[Z^{(1)}_{pp'} Z^{(2)}_{pp'} + Z^{(1)}_{pp'} Z^{(0)}_{pp'} \right]}{Z^{(0)}_{pp'} Z^{(1)}_{pp'} + Z^{(1)}_{pp'} Z^{(2)}_{pp'} + Z^{(2)}_{pp'} Z^{(0)}_{pp'}}$$

$$V_a^{(1)} = V_a^{(2)} = V_a^{(0)} = \frac{I_{mn}\left[Z^{(1)}_{pp'} Z^{(2)}_{pp'} + Z^{(1)}_{pp'} Z^{(0)}_{pp'}\right]}{Z^{(0)}_{pp'} Z^{(1)}_{pp'} + Z^{(1)}_{pp'} Z^{(2)}_{pp'} + Z^{(2)}_{pp'} Z^{(0)}_{pp'}}$$

$$V_a^{(1)} = V_a^{(2)} = V^{(0)} = \frac{I_{mn}\, Z^{(1)}_{pp'}\, Z^{(2)}_{pp'}}{Z^{(1)}_{pp'} + Z^{(2)}_{pp'}} = \frac{(0.6\angle{-36.87^\circ})(j0.712)(j0.712)}{j0.712 + j0.712} = 0.2136\angle 53.13^\circ$$

Current Injections @ Bus m

Positive $\frac{V^{(1)}}{Z^{(1)}}$ Negative $\frac{V^{(2)}}{Z^{(2)}}$ Zero $\frac{V^{(0)}}{Z^{(0)}}$

Current Injections @ Bus n

Positive $-\frac{V^{(1)}}{Z^{(1)}}$ Negative $-\frac{V^{(2)}}{Z^{(2)}}$ Zero $-\frac{V^{(0)}}{Z^{(1)}}$

Changes In Voltages

$$\Delta V^{(1)} = \left[-\frac{V^{(1)}}{Z^{(1)}}\right]\left[Z_{nn} - Z_{nm}\right] = \left[\frac{V^{(1)}}{Z^{(1)}}\right]\left[Z_{nm}^{(1)} - Z_{nn}^{(1)}\right]$$

$$\Delta V^{(0)} = \left[\frac{V^{(0)}}{Z^{(0)}}\right]\left[Z_{nm}^{(0)} - Z_{nn}^{(0)}\right]$$

$$\Delta V^{(2)} = \left[\frac{V^{(2)}}{Z^{(2)}}\right]\left[Z_m^{(2)} - Z_{nn}^{(2)}\right]$$

$$\Delta V^{(1)} = \Delta V^{(2)} = \left[\frac{0.2136\angle 53.13^\circ}{j0.15}\right]\left[j0.1104 - j0.1696\right] = 0.0843\angle{-126.87^\circ}$$

$$\Delta V^{(0)} = \left[\frac{0.2136\angle 53.13^\circ}{j0.50}\right]\left[j0.08 - j0.58\right] = 0.2136\angle{-126.87^\circ}$$

$$\Delta V_3 = \Delta V_3^{(1)} + \Delta V_3^{(2)} + \Delta V_3^{(0)} = \overset{(1)}{(0.0843\angle{-126.87^\circ})} + \overset{(2)}{(0.0843\angle{-126.87^\circ})} + \overset{(0)}{(0.2136\angle{-126.87^\circ})}$$

$$\Delta V_3 = 0.3822\angle{-126.87^\circ}$$

Therefore the voltage after the fault:

$$V^{NEW} = V_3^{PREFAULT} + \Delta V_3 = (1.0\angle 0^\circ) + (0.3822\angle{-126.87^\circ}) = 0.829\angle{-21.64^\circ} \text{ p.u.}$$

$$V^{NEW} = (0.829)(345\text{ kV}) = \boxed{286\text{ kV}}$$